Quantifying Electron Tunneling Behaviors in Esaki Diodes

Enoch Kang

Southwest Center for Education And The Natural Environment

Abstract

An Esaki diode generates less current with increasing temperature at relatively low biases while it generates more current with increasing temperature at higher biases. This project provides an explanation for this unusual phenomenon by studying the difference between the current-generating processes of the Esaki diode at high and low temperatures.

Background

Electron tunneling is a quantum mechanical phenomenon where an electron "disappears" from one side of a thin barrier and instantaneously "appears" on the other side without an input of energy to "surmount" the barrier (General Electric 7).

Leo Esaki discovered the tunneling effects in a semiconductor p-n junction and invented a diode that generates current using the tunneling principle, which was named "Esaki diode" after his own name (Lee, Easter and Bell 3).

From Last Year's Project

Figure 1 shows two IV characterizations of the Esaki diode performed last year.

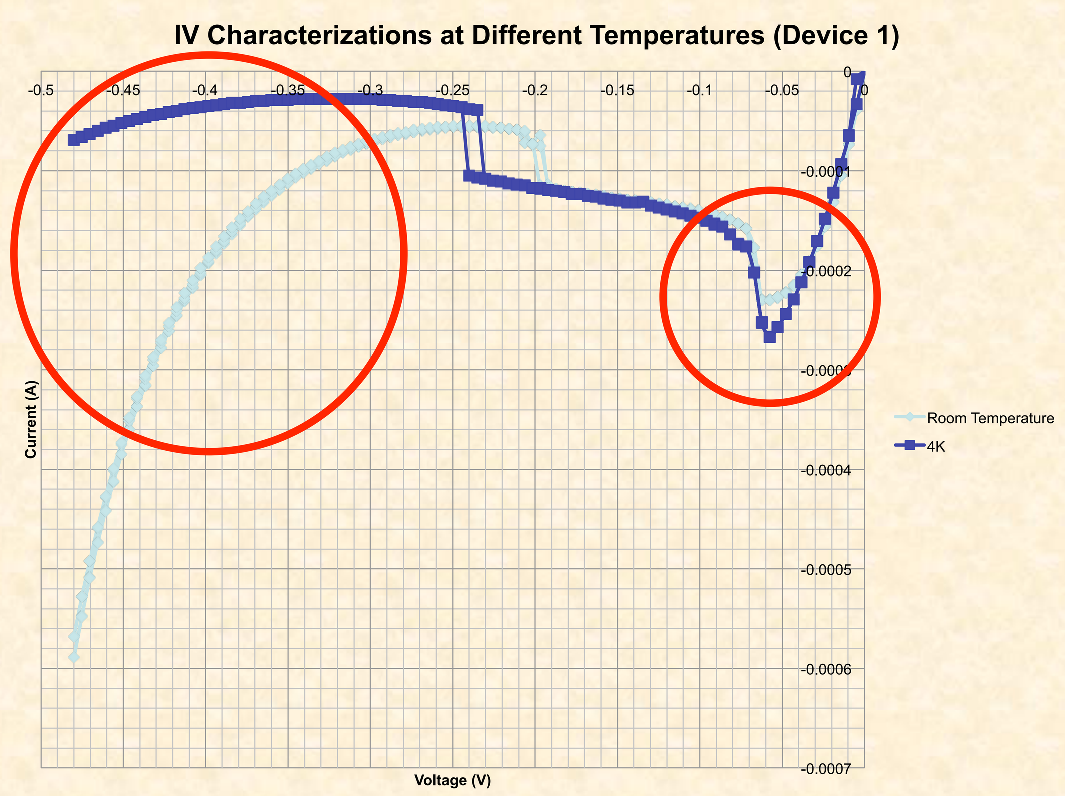

Figure 1. IV Characterizations of an Esaki diode at 4K and 298K (from my last year's project)

The regions with the red circles are the areas of interest here.

Under a low bias, the diode produced more current at a lower temperature. On the other hand, under a high bias, more current was generated at a higher temperature.

The question is: *why does this happen?*

Hypothesis

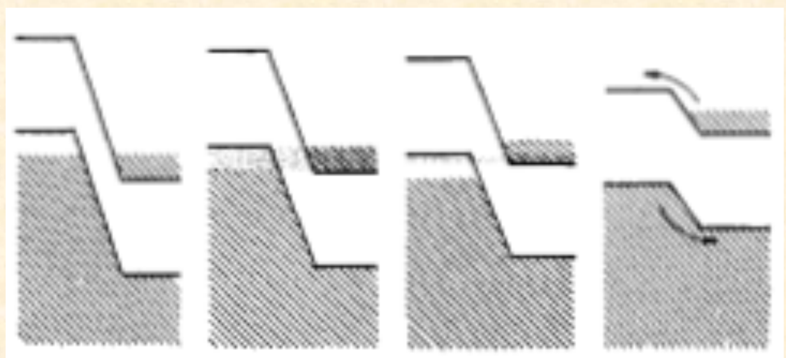

Figure 2. Band diagrams of the Esaki diode with increasing bias (Hall 4).

Looking at Figure 2, one can see that at low biases, the diode generates current with tunneling where electrons "tunnel through" the barrier (1-3) while it generates current conventionally at high biases where electrons go "over" the barrier (4).

I hypothesized that this is the cause of the phenomenon observed last year:

More tunneling occurs with decreasing temperature while "surmounting" the barrier occurs more easily at higher temperatures.

Literature Review

Because how higher bias leads to higher current in the conventional way is a well-understood process, I focused on how tunneling may lead to more current at low temperatures.

With the electron tunneling at a p-n junction, the current is generated by electrons on the *n* side tunneling through the barrier and filling in the holes on the *p* side.

Therefore, the amount of electron tunneling taking place could be quantified as the number of electrons on the n side n_i multiplied by the number of holes the p side p_i.

The Formula

n_i is calculated as the product of probability of having an electron at a given energy level, $P_e(E)$, and the number of allowed energy levels on the conduction band, $N_c(E)$ (Smith 80).

p_i is calculated as the product of probability of having a hole at a given energy level, $P_h(E)$, and the number of allowed energy levels on the valence band, $N_v(E)$ (Smith 80).

So the following equation is established:

$$n_i \cdot p_i = P_e(E)N_c(E) \cdot P_h(E)N_v(E)$$

Explanation

$P_e(E)$ is calculated as the following:

$$P_e(E) = \frac{1}{e^{(E-E_{cf})/(kT)}+1} \quad \text{(Smith 77).}$$

E is the energy level given, E_{cf} is the Fermi level of the conduction band, (compared to E_{vf} of the valence band) k is the Boltzmann constant, and T is the temperature.

$P_h(E)$ is calculated as the following:

$$P_h(E) = \frac{1}{e^{(E_{vf}-E)/(kT)}+1} \quad \text{(Smith 79).}$$

Notice that $P_h(E)$ is just a simplification of $1-P_e(E)$, which is logical because what is not occupied by an electron is occupied by a hole at a given energy level.

$N_c(E)$ is defined as:

$$N_c(E) = 2\pi(2m_e)^{\frac{3}{2}}h^{-3}(E-E_{be}) \quad \text{(Smith 81).}$$

m_e is the effective mass of an electron, h is the Planck constant, and E_{be} is the lowest energy level occupied by electrons (Recall that E_{vf} refers to the lowest energy level occupied by holes).

$N_v(E)$ is similarly defined as:

$$N_v(E) = 2\pi(2m_h)^{\frac{3}{2}}h^{-3}(E_{th}-E) \quad \text{(Smith 81).}$$

m_h is the effective mass of a hole, h is the Planck constant, and E_{th} is the highest energy level occupied by holes (Recall that E_{cf} refer s to the highest energy level occupied by electrons).

Fermi-Dirac Distribution

The equation for $P_e(E)$ is also called as the Fermi-Dirac distribution function. It is best understood as an equation showing the probability of occupancy of an energy level by electrons that obey the Pauli exclusion theory (Zeghbroeck par. 2).

At absolute zero, the energy levels below the Fermi level are completely filled and no levels above the level are filled. However, at higher temperatures, the Fermi level is more gradual rather than abrupt because some electrons are excited over the Fermi level, leaving holes behind (Zeghbroeck par. 2)

How This Affects Tunneling

Figure 3 shows a diagram of a typical tunneling with an "abrupt" Fermi level at a p-n junction.

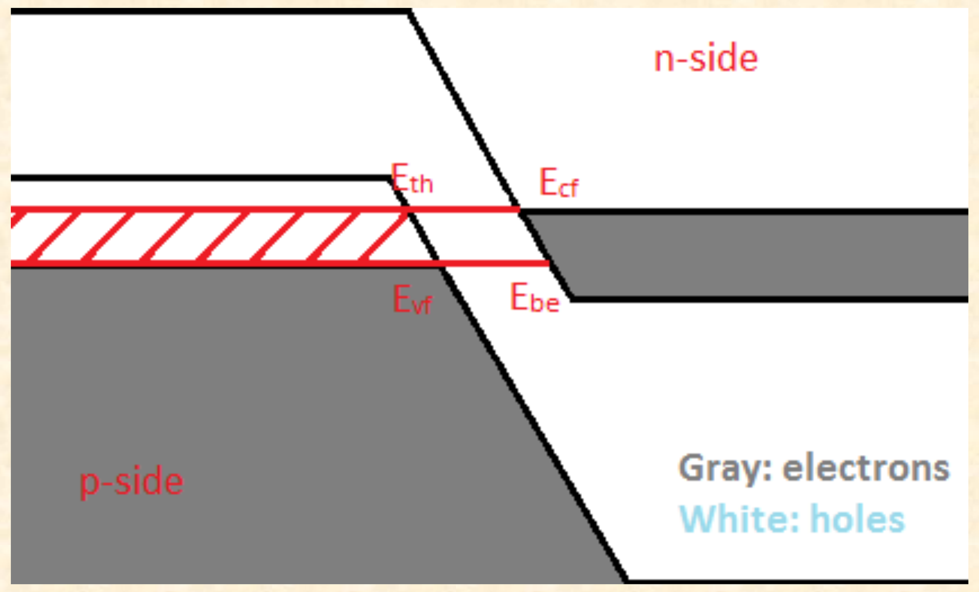

Figure 3. Band diagram of a p-n junction (created by myself)

Notice that tunneling only takes place between E_{vf} and E_{fe}. However, according to the Fermi-Dirac distribution, this is only possible at absolute zero.

Figure 4 shows the difference between tunneling behaviors at a p-n junction at three different temperatures.

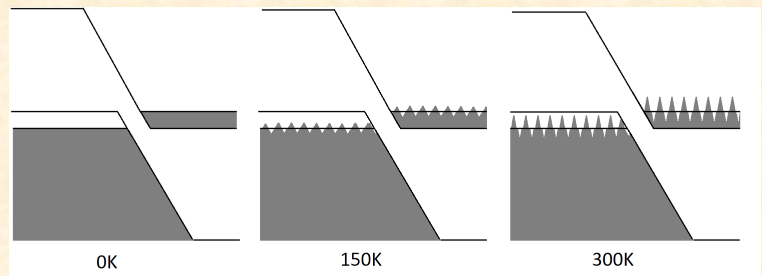

Figure 4. Tunneling behaviors with bias(created by myself)

Notice that lower temperature leads to more aligned electrons and holes, which means more tunneling. *This explains last year's data!*

Integration

From Figure 4, notice that tunneling actually takes place between E_{be} and E_{th}. So, in order to quantify the amount of tunneling at all energy levels at a given temperature, following integral can be established:

$$\int_{E_{be}}^{E_{th}} \frac{4\pi}{h^3}(2m_e)^{\frac{3}{2}}(E-E_{be})^{\frac{1}{2}} \times \frac{1}{1+e^{(E-E_{cf})/(kT)}} \times \frac{4\pi}{h^3}(2m_h)^{\frac{3}{2}}(E_{th}-E)^{\frac{1}{2}} \times \frac{1}{1+e^{(E_{vf}-E)/(kT)}}\,dE$$

Removing the constants, the integral becomes:

$$\int_{E_{be}}^{E_{th}} (E-E_{be})^{\frac{1}{2}} \times \frac{1}{1+e^{(E-E_{cf})/(kT)}} \times (E_{th}-E)^{\frac{1}{2}} \times \frac{1}{1+e^{(E_{vf}-E)/(kT)}}\,dE$$

Taking into account the tunneling occurring in the reverse direction, the reduced integral becomes:

$$\int_{E_{be}}^{E_{th}} (E-E_{be})^{\frac{1}{2}} \times \frac{1}{1+e^{(E-E_{cf})/(kT)}} \times (E_{th}-E)^{\frac{1}{2}} \times \frac{1}{1+e^{(E_{vf}-E)/(kT)}} - (E-E_{be})^{\frac{1}{2}} \times \frac{1}{1+e^{(E_{cf}-E)/(kT)}} \times (E_{th}-E)^{\frac{1}{2}} \times \frac{1}{1+e^{(E-E_{vf})/(kT)}}\,dE$$

Assuming the band width to be 0.06V, the integral was calculated for three different temperature values: 4K, 150K, and 300K. The following graph shows the results:

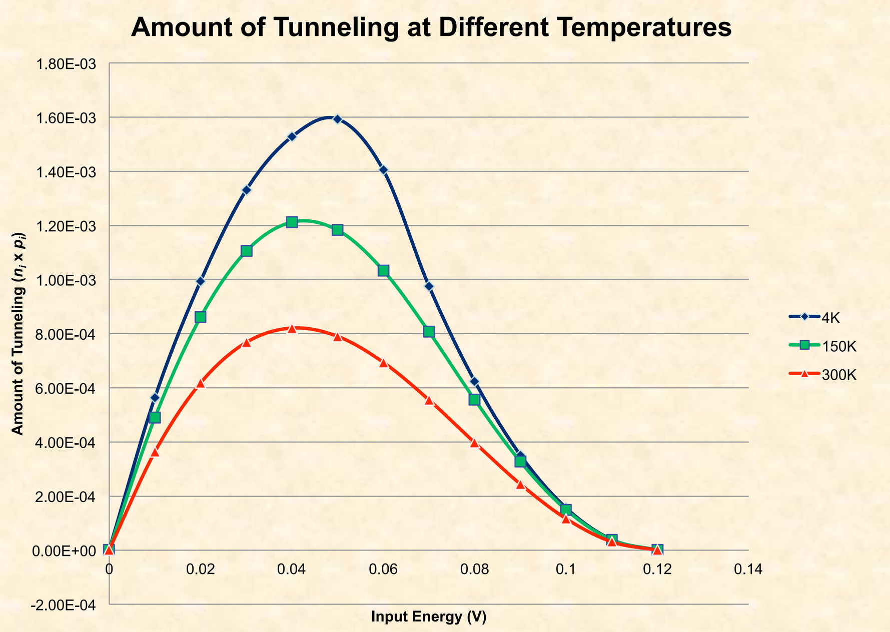

Figure 5. Amount of tunneling at different temperatures (created by myself)

Conclusion

A theoretical explanation confirms last year's data and my hypothesis. It is the tendency of electrons and holes to "fluctuate" over the Fermi level at non-zero temperatures that enables higher generation of current at lower temperatures.

My project leads to a more accurate understanding of how the Esaki diode behaves at low temperature. This can be practical to design future products with the diode, including oscillators. Also, it expanded the knowledge humans have on tunneling behaviors in general, which hopefully scientists and engineers can take advantage of in the future for more practical uses.

References

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